

# Non Singular Origin of the Universe and its Present Vacuum Energy Density

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## Abstract

We consider a non singular origin for the Universe starting from an Einstein static Universe, the so called "emergent universe" scenario, in the framework of a theory which uses two volume elements  $\sqrt{-g}d^4x$  and  $\Phi d^4x$ , where  $\Phi$  is a metric independent density, used as an additional measure of integration. Also curvature, curvature square terms and for scale invariance a dilaton field  $\phi$  are considered in the action. The first order formalism is applied. The integration of the equations of motion associated with the new measure gives rise to the spontaneous symmetry breaking (S.S.B) of scale invariance (S.I.). After S.S.B. of S.I., it is found that a non trivial potential for the dilaton is generated. In the Einstein frame we also add a cosmological term that parametrizes the zero point fluctuations. The resulting effective potential for the dilaton contains two flat regions, for  $\phi \rightarrow \infty$  relevant for the non singular origin of the Universe, followed by an inflationary phase and  $\phi \rightarrow -\infty$ , describing our present Universe. The dynamics of the scalar field becomes non linear and these non linearities are instrumental in the stability of some of the emergent universe solutions, which exists for a parameter range of values of the vacuum energy in  $\phi \rightarrow -\infty$ , which must be positive but not very big, avoiding the extreme fine tuning required to keep the vacuum energy density of the present universe small. Zero vacuum energy density for the present universe defines the threshold for the creation of the universe.

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## I. INTRODUCTION

One of the most important and intriguing issues of modern physics is the so called "Cosmological Constant Problem" [1], [2],[3] (CCP), most easily seen by studying the apparently uncontrolled behavior of the zero point energies, which would lead to a corresponding equally uncontrolled vacuum energy or cosmological constant term. Even staying at the classical level, the observed very small cosmological term in the present universe is still very puzzling.

One point of view to the CCP that has been popular has been to provide a bound based on the "anthropic principle" [4]. In this approach, a too large Cosmological Constant will not provide the necessary conditions required for the existence of life, the anthropic principle provides then an upper bound on the cosmological constant.

One problem with this approach is for example that it relies on our knowledge of life as we know it and ignores the possibility that other life forms could be possible, for which other (unknown) bounds would be relevant, therefore the reasoning appears by its very nature subjective, since of course if the observed cosmological constant will be different, our universe will be different and this could include different kind of life that may be could have adjusted itself to a higher cosmological constant of the universe. But even accepting the validity of anthropic considerations, we still do not understand why the observed vacuum energy density must be positive instead of possibly a very small negative quantity. Accepting the anthropic explanation means may be also giving up on discovering important physics related to the CCP and this may be the biggest objection.

Nevertheless, the idea of associating somehow restrictions on the origin of the universe with the cosmological constant problem seems interesting. We will take on this point of view, but leave out the not understood concept of life out from our considerations. Instead, we will require, in a very specific framework, the non singular origin of the universe. The advantage of this point of view is that it is formulated in terms of ideas of physics alone, without reference to biology, which unlike physics, has not reached the level of an exact science. Another interesting consequence is that we can learn that a non singularly created universe may not have a too big cosmological constant, an effect that points to a certain type of gravitational suppression of UV divergences in quantum field theory.

In this respect, one should point out that even in the context of the inflationary scenario [5], [6], [7], [8], which solves many cosmological problems, one still encounters the initial

singularity problem which remains unsolved, showing that the universe necessarily had a singular beginning for generic inflationary cosmologies [9], [10], [11], [12], [13].

Here we will adopt the very attractive "Emergent Universe" scenario, where those conclusions concerning singularities can be avoided [14],[15], [16], [17], [18], [19], [20], [21]. The way to escape the singularity in these models is to violate the geometrical assumptions of these theorems, which assume i) that the universe has open space sections ii) the Hubble expansion is always greater than zero in the past. In [14],[15] the open space section condition is violated since closed Robertson Walker universes with  $k = 1$  are considered and the Hubble expansion can become zero, so that both i) and ii) are avoided.

In [14], [15] even models based on standard General Relativity, ordinary matter and minimally coupled scalar fields were considered and can provide indeed a non singular (geodesically complete) inflationary universe, with a past eternal Einstein static Universe that eventually evolves into an inflationary Universe.

Those most simple models suffer however from instabilities, associated with the instability of the Einstein static universe. The instability is possible to cure by going away from GR, considering non perturbative corrections to the Einstein's field equations in the context of the loop quantum gravity[16], a brane world cosmology [17], considering the Starobinski model for radiative corrections (which cannot be derived from an effective action)[18] or exotic matter[19]. In addition to this, the consideration of a Jordan Brans Dicke model also can provide a stable initial state for the emerging universe scenario [20], [21].

In this paper we propose a different theoretical framework where such emerging universe scenario is realized in a natural way, where instabilities are avoided and a succesfull inflationary phase with a gracefull exit can be achieved. The model we will use was studied first in [22], however, we differ with [22] in our choice of the state (with a lower vacuum energy density) that best represents the present state of the universe. This is crucial, since as it should be obvious, the discussion of the CCP depends crucially on what vacuum we take. We will express the stability and existence conditions for the non singular universe in terms of the energy of the vacuum of our candidate for the present Universe. We will also by the way discuss and correct a few typos in [22] and improve a bit the discussion of some notions discussed there as well.

We work in the context of a theory built along the lines of the two measures theory (TMT) [23]-[40], [41], [42]-[44], [47]-[51], [52] and more specifically in the context of the scale

invariant realization of such theories [41], [42]-[46], [47]-[51], [52]. These theories can provide a new approach to the cosmological constant problem and can be generalized to obtain also a theory with a dynamical spacetime [54] . We will consider a slight generalization of the TMT case, where, we consider also the possible effects of zero point energy densities, thus "softly breaking" the basic structure of TMT for this purpose. We will show how the stated goals of a stable emerging universe can be achieved in the framework of the model and also how the stability of the emerging universe imposes interesting constraints on the energy density of the ground state of the theory as defined in this paper: it must be positive but not very large, thus the vacuum energy and therefore the term that softly breaks the TMT structure appears to be naturally controlled.

The paper will be organized as follows: First we review the principles of the TMT and in particular the model studied in [41], which has global scale invariance and how this can be the basis for the emerging universe. Such model gives rise, in the effective Einstein frame, to an effective potential for a dilaton field (needed to implement an interesting model with global scale invariance) which has a flat region. Following this, we look at the generalization of this model [52] by adding a curvature square or simply " $R^2$  term" and show that the resulting model contains now two flat regions. The existence of two flat regions for the potential is shown to be consequence of the s.s.b. of the scale symmetry. We then consider the incorporation in the model of the zero point fluctuations, parametrized by a cosmological constant in the Einstein frame. In this resulting model, there are two possible types of emerging universe solutions, for one of those, the initial Einstein Universe can be stabilized due to the nonlinearities of the model, provided the vacuum energy density of the ground state is positive but not very large. This is a very satisfactory results, since it means that the stability of the emerging universe prevents the vacuum energy in the present universe from being very large!. The transition from the emergent universe to the ground state goes through an intermediate inflationary phase, therefore reproducing the basic standard cosmological model as well. We end with a discussion section and present the point of view that the creation of the universe can be considered as a "threshold event" for zero present vacuum energy density, which naturally gives a positive but small vacuum energy density.

## II. INTRODUCING A NEW MEASURE

The general structure of general coordinate invariant theories is taken usually as

$$S_1 = \int L_1 \sqrt{-g} d^4x \quad (1)$$

where  $g = \det(g_{\mu\nu})$ . The introduction of  $\sqrt{-g}$  is required since  $d^4x$  by itself is not a scalar but the product  $\sqrt{-g}d^4x$  is a scalar. Inserting  $\sqrt{-g}$ , which has the transformation properties of a density, produces a scalar action  $S_1$ , as defined by eq.(1), provided  $L_1$  is a scalar.

In principle nothing prevents us from considering other densities instead of  $\sqrt{-g}$ . One construction of such alternative "measure of integration", is obtained as follows: given 4-scalars  $\varphi_a$  ( $a = 1,2,3,4$ ), one can construct the density

$$\Phi = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\mu \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d \quad (2)$$

and consider in addition to the action  $S_1$ , as defined by eq.(1),  $S_2$ , defined as

$$S_2 = \int L_2 \Phi d^4x \quad (3)$$

$L_2$  is again some scalar, which may contain the curvature (i.e. the gravitational contribution) and a matter contribution, as it can be the case for  $S_1$ , as defined by eq.(1).

In the action  $S_2$  defined by eq.(3) the measure carries degrees of freedom independent of that of the metric and that of the matter fields. The most natural and successful formulation of the theory is achieved when the connection is also treated as an independent degree of freedom. This is what is usually referred to as the first order formalism.

One can consider both contributions, and allowing therefore both geometrical objects to enter the theory and take as our action

$$S = \int L_1 \sqrt{-g} d^4x + \int L_2 \Phi d^4x \quad (4)$$

Here  $L_1$  and  $L_2$  are  $\varphi_a$  independent.

We will study now the dynamics of a scalar field  $\phi$  interacting with gravity as given by the following action, where except for the potential terms  $U$  and  $V$  we have conformal invariance, the potential terms  $U$  and  $V$  break down this to global scale invariance.

$$S_L = \int L_1 \sqrt{-g} d^4x + \int L_2 \Phi d^4x \quad (5)$$

$$L_1 = U(\phi) \quad (6)$$

$$L_2 = \frac{-1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \quad (7)$$

$$R(\Gamma, g) = g^{\mu\nu} R_{\mu\nu}(\Gamma), R_{\mu\nu}(\Gamma) = R_{\mu\nu\lambda}^\lambda \quad (8)$$

$$R_{\mu\nu\sigma}^\lambda(\Gamma) = \Gamma_{\mu\nu,\sigma}^\lambda - \Gamma_{\mu\sigma,\nu}^\lambda + \Gamma_{\alpha\sigma}^\lambda \Gamma_{\mu\nu}^\alpha - \Gamma_{\alpha\nu}^\lambda \Gamma_{\mu\sigma}^\alpha. \quad (9)$$

The suffix  $L$  in  $S_L$  is to emphasize that here the curvature appears only linearly. Here, except for the potential terms  $U$  and  $V$  we have conformal invariance, the potential terms  $U$  and  $V$  break down this to global scale invariance. Since the breaking of local conformal invariance is only through potential terms, we call this a "soft breaking".

In the variational principle  $\Gamma_{\mu\nu}^\lambda, g_{\mu\nu}$ , the measure fields scalars  $\varphi_a$  and the "matter" - scalar field  $\phi$  are all to be treated as independent variables although the variational principle may result in equations that allow us to solve some of these variables in terms of others.

For the case the potential terms  $U = V = 0$  we have local conformal invariance

$$g_{\mu\nu} \rightarrow \Omega(x) g_{\mu\nu} \quad (10)$$

and  $\varphi_a$  is transformed according to

$$\varphi_a \rightarrow \varphi'_a = \varphi'_a(\varphi_b) \quad (11)$$

$$\Phi \rightarrow \Phi' = J(x) \Phi \quad (12)$$

where  $J(x)$  is the Jacobian of the transformation of the  $\varphi_a$  fields.

This will be a symmetry in the case  $U = V = 0$  if

$$\Omega = J \quad (13)$$

Notice that  $J$  can be a local function of space time, this can be arranged by performing for the  $\varphi_a$  fields one of the (infinite) possible diffeomorphisms in the internal  $\varphi_a$  space.

We can still retain a global scale invariance in model for very special exponential form for the  $U$  and  $V$  potentials. Indeed, if we perform the global scale transformation ( $\theta = \text{constant}$ )

$$g_{\mu\nu} \rightarrow e^\theta g_{\mu\nu} \quad (14)$$

then (9) is invariant provided  $V(\phi)$  and  $U(\phi)$  are of the form [41]

$$V(\phi) = f_1 e^{\alpha\phi}, U(\phi) = f_2 e^{2\alpha\phi} \quad (15)$$

and  $\varphi_a$  is transformed according to

$$\varphi_a \rightarrow \lambda_{ab} \varphi_b \quad (16)$$

which means

$$\Phi \rightarrow \det(\lambda_{ab}) \Phi \equiv \lambda \Phi \quad (17)$$

such that

$$\lambda = e^\theta \quad (18)$$

and

$$\phi \rightarrow \phi - \frac{\theta}{\alpha}. \quad (19)$$

We will now work out the equations of motion after introducing  $V(\phi)$  and  $U(\phi)$  and see how the integration of the equations of motion allows the spontaneous breaking of the scale invariance.

Let us begin by considering the equations which are obtained from the variation of the fields that appear in the measure, i.e. the  $\varphi_a$  fields. We obtain then

$$A_a^\mu \partial_\mu L_2 = 0 \quad (20)$$

where  $A_a^\mu = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d$ . Since it is easy to check that  $A_a^\mu \partial_\mu \varphi_{a'} = \frac{\delta a a'}{4} \Phi$ , it follows that  $\det(A_a^\mu) = \frac{4^{-4}}{4!} \Phi^3 \neq 0$  if  $\Phi \neq 0$ . Therefore if  $\Phi \neq 0$  we obtain that  $\partial_\mu L_2 = 0$ , or that

$$L_2 = \frac{-1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V = M \quad (21)$$

where  $M$  is constant. Notice that this equation breaks spontaneously the global scale invariance of the theory, since the left hand side has a non trivial transformation under the scale transformations, while the right hand side is equal to  $M$ , a constant that after we integrate the equations is fixed, cannot be changed and therefore for any  $M \neq 0$  we have obtained indeed, spontaneous breaking of scale invariance.

We will see what is the connection now. As we will see, the connection appears in the original frame as a non Riemannian object. However, we will see that by a simple conformal tranformation of the metric we can recover the Riemannian structure. The interpretation of the equations in the frame gives then an interesting physical picture, as we will see.

Let us begin by studying the equations obtained from the variation of the connections  $\Gamma_{\mu\nu}^\lambda$ . We obtain then

$$-\Gamma_{\mu\nu}^\lambda - \Gamma_{\beta\mu}^\alpha g^{\beta\lambda} g_{\alpha\nu} + \delta_\nu^\lambda \Gamma_{\mu\alpha}^\alpha + \delta_\mu^\lambda g^{\alpha\beta} \Gamma_{\alpha\beta}^\gamma g_{\gamma\nu} - g_{\alpha\nu} \partial_\mu g^{\alpha\lambda} + \delta_\mu^\lambda g_{\alpha\nu} \partial_\beta g^{\alpha\beta} - \delta_\nu^\lambda \frac{\Phi_{,\mu}}{\Phi} + \delta_\mu^\lambda \frac{\Phi_{,\nu}}{\Phi} = 0 \quad (22)$$

If we define  $\Sigma_{\mu\nu}^\lambda$  as  $\Sigma_{\mu\nu}^\lambda = \Gamma_{\mu\nu}^\lambda - \{\lambda_{\mu\nu}\}$  where  $\{\lambda_{\mu\nu}\}$  is the Christoffel symbol, we obtain for  $\Sigma_{\mu\nu}^\lambda$  the equation

$$-\sigma_{,\lambda} g_{\mu\nu} + \sigma_{,\mu} g_{\nu\lambda} - g_{\nu\alpha} \Sigma_{\lambda\mu}^\alpha - g_{\mu\alpha} \Sigma_{\nu\lambda}^\alpha + g_{\mu\nu} \Sigma_{\lambda\alpha}^\alpha + g_{\nu\lambda} g_{\alpha\mu} g^{\beta\gamma} \Sigma_{\beta\gamma}^\alpha = 0 \quad (23)$$

where  $\sigma = \ln \chi$ ,  $\chi = \frac{\Phi}{\sqrt{-g}}$ .

The general solution of eq.(24) is

$$\Sigma_{\mu\nu}^\alpha = \delta_\mu^\alpha \lambda_{,\nu} + \frac{1}{2} (\sigma_{,\mu} \delta_\nu^\alpha - \sigma_{,\beta} g_{\mu\nu} g^{\alpha\beta}) \quad (24)$$

where  $\lambda$  is an arbitrary function due to the  $\lambda$  - symmetry of the curvature [55]  $R_{\mu\nu\alpha}^\lambda(\Gamma)$ ,

$$\Gamma_{\mu\nu}^\alpha \rightarrow \Gamma_{\mu\nu}'^\alpha = \Gamma_{\mu\nu}^\alpha + \delta_\mu^\alpha Z_{,\nu} \quad (25)$$

$Z$  being any scalar (which means  $\lambda \rightarrow \lambda + Z$ ).

If we choose the gauge  $\lambda = \frac{\sigma}{2}$ , we obtain

$$\Sigma_{\mu\nu}^\alpha(\sigma) = \frac{1}{2} (\delta_\mu^\alpha \sigma_{,\nu} + \delta_\nu^\alpha \sigma_{,\mu} - \sigma_{,\beta} g_{\mu\nu} g^{\alpha\beta}). \quad (26)$$

Considering now the variation with respect to  $g^{\mu\nu}$ , we obtain

$$\Phi \left( \frac{-1}{\kappa} R_{\mu\nu}(\Gamma) + \frac{1}{2} \phi_{,\mu} \phi_{,\nu} \right) - \frac{1}{2} \sqrt{-g} U(\phi) g_{\mu\nu} = 0 \quad (27)$$

solving for  $R = g^{\mu\nu} R_{\mu\nu}(\Gamma)$  from eq.(27) and introducing in eq.(21), we obtain

$$M + V(\phi) - \frac{2U(\phi)}{\chi} = 0 \quad (28)$$

a constraint that allows us to solve for  $\chi$ ,

$$\chi = \frac{2U(\phi)}{M + V(\phi)}. \quad (29)$$

To get the physical content of the theory, it is best consider variables that have well defined dynamical interpretation. The original metric does not has a non zero canonical momenta. The fundamental variable of the theory in the first order formalism is the connection and its canonical momenta is a function of  $\bar{g}_{\mu\nu}$ , given by,



$$\bar{g}_{\mu\nu} = \chi g_{\mu\nu} \quad (30)$$

and  $\chi$  given by eq.(29). Interestingly enough, working with  $\bar{g}_{\mu\nu}$  is the same as going to the "Einstein Conformal Frame". In terms of  $\bar{g}_{\mu\nu}$  the non Riemannian contribution  $\Sigma_{\mu\nu}^\alpha$  disappears from the equations. This is because the connection can be written as the Christoffel symbol of the metric  $\bar{g}_{\mu\nu}$ . In terms of  $\bar{g}_{\mu\nu}$  the equations of motion for the metric can be written then in the Einstein form (we define  $\bar{R}_{\mu\nu}(\bar{g}_{\alpha\beta})$  = usual Ricci tensor in terms of the bar metric =  $R_{\mu\nu}$  and  $\bar{R} = \bar{g}^{\mu\nu}\bar{R}_{\mu\nu}$  )

$$\bar{R}_{\mu\nu}(\bar{g}_{\alpha\beta}) - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R}(\bar{g}_{\alpha\beta}) = \frac{\kappa}{2}T_{\mu\nu}^{eff}(\phi) \quad (31)$$

where

$$T_{\mu\nu}^{eff}(\phi) = \phi_{,\mu}\phi_{,\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\phi_{,\alpha}\phi_{,\beta}\bar{g}^{\alpha\beta} + \bar{g}_{\mu\nu}V_{eff}(\phi) \quad (32)$$

and

$$V_{eff}(\phi) = \frac{1}{4U(\phi)}(V + M)^2. \quad (33)$$

In terms of the metric  $\bar{g}^{\alpha\beta}$ , the equation of motion of the Scalar field  $\phi$  takes the standard General - Relativity form

$$\frac{1}{\sqrt{-\bar{g}}}\partial_\mu(\bar{g}^{\mu\nu}\sqrt{-\bar{g}}\partial_\nu\phi) + V'_{eff}(\phi) = 0. \quad (34)$$

Notice that if  $V + M = 0$ ,  $V_{eff} = 0$  and  $V'_{eff} = 0$  also, provided  $V'$  is finite and  $U \neq 0$  there. This means the zero cosmological constant state is achieved without any sort of fine tuning. That is, independently of whether we add to  $V$  a constant piece, or whether we change the value of  $M$ , as long as there is still a point where  $V + M = 0$ , then still  $V_{eff} = 0$  and  $V'_{eff} = 0$  ( still provided  $V'$  is finite and  $U \neq 0$  there). This is the basic feature that characterizes the TMT and allows it to solve the 'old' cosmological constant problem, at least at the classical level.

In what follows we will study the effective potential (33) for the special case of global scale invariance, which as we will see displays additional very special features which makes it attractive in the context of cosmology.

Notice that in terms of the variables  $\phi, \bar{g}_{\mu\nu}$ , the "scale" transformation becomes only a shift in the scalar field  $\phi$ , since  $\bar{g}_{\mu\nu}$  is invariant (since  $\chi \rightarrow \lambda^{-1}\chi$  and  $g_{\mu\nu} \rightarrow \lambda g_{\mu\nu}$ )

$$\bar{g}_{\mu\nu} \rightarrow \bar{g}_{\mu\nu}, \phi \rightarrow \phi - \frac{\theta}{\alpha}. \quad (35)$$

If  $V(\phi) = f_1 e^{\alpha\phi}$  and  $U(\phi) = f_2 e^{2\alpha\phi}$  as required by scale invariance eqs.(14),(16),(17),(18),(19), we obtain from the expression (33)

$$V_{eff} = \frac{1}{4f_2}(f_1 + M e^{-\alpha\phi})^2 \quad (36)$$

Since we can always perform the transformation  $\phi \rightarrow -\phi$  we can choose by convention  $\alpha > 0$ . We then see that as  $\phi \rightarrow \infty$ ,  $V_{eff} \rightarrow \frac{f_1^2}{4f_2} = \text{const.}$  providing an infinite flat region. Also a minimum is achieved at zero cosmological constant for the case  $\frac{f_1}{M} < 0$  at the point

$$\phi_{min} = \frac{-1}{\alpha} \ln \left| \frac{f_1}{M} \right|. \quad (37)$$

In conclusion, the scale invariance of the original theory is responsible for the non appearance (in the physics) of a certain scale, that associated to  $M$ . However, masses do appear, since the coupling to two different measures of  $L_1$  and  $L_2$  allow us to introduce two independent couplings  $f_1$  and  $f_2$ , a situation which is unlike the standard formulation of globally scale invariant theories, where usually no stable vacuum state exists.

The constant of integration  $M$  plays a very important role indeed: any non vanishing value for this constant implements, already at the classical level S.S.B. of scale invariance.

### III. GENERATION OF TWO FLAT REGIONS AFTER THE INTRODUCTION OF A $R^2$ TERM

As we have seen, it is possible to obtain a model that through a spontaneous breaking of scale invariance can give us a flat region. We want to obtain now two flat regions in our effective potential. A simple generalization of the action  $S_L$  will fix this. What one needs to do is simply consider the addition of a scale invariant term of the form

$$S_{R^2} = \epsilon \int (g^{\mu\nu} R_{\mu\nu}(\Gamma))^2 \sqrt{-g} d^4x \quad (38)$$

The total action being then  $S = S_L + S_{R^2}$ . In the first order formalism  $S_{R^2}$  is not only globally scale invariant but also locally scale invariant, that is conformally invariant (recall that in the first order formalism the connection is an independent degree of freedom and it does not transform under a conformal transformation of the metric). The higher curvature theories in the context of the second order formalism [60]-[65],[66]- [68] have a completely

different behavior, giving higher order equations, etc., unlike higher curvature theories in the context of the first order formalism, like we do here.

Let us see what the equations of motion tell us, now with the addition of  $S_{R^2}$  to the action. First of all, since the addition has been only to the part of the action that couples to  $\sqrt{-g}$ , the equations of motion derived from the variation of the measure fields remains unchanged. That is eq.(21) remains valid.

The variation of the action with respect to  $g^{\mu\nu}$  gives now

$$R_{\mu\nu}(\Gamma)(\frac{-\Phi}{\kappa} + 2\epsilon R\sqrt{-g}) + \Phi \frac{1}{2}\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}(\epsilon R^2 + U(\phi))\sqrt{-g}g_{\mu\nu} = 0 \quad (39)$$

It is interesting to notice that if we contract this equation with  $g^{\mu\nu}$ , the  $\epsilon$  terms do not contribute. This means that the same value for the scalar curvature  $R$  is obtained as in section II, if we express our result in terms of  $\phi$ , its derivatives and  $g^{\mu\nu}$ . Solving the scalar curvature from this and inserting in the other  $\epsilon$  - independent equation  $L_2 = M$  we get still the same solution for the ratio of the measures which was found in the case where the  $\epsilon$  terms were absent, i.e.  $\chi = \frac{\Phi}{\sqrt{-g}} = \frac{2U(\phi)}{M+V(\phi)}$ .

In the presence of the  $\epsilon R^2$  term in the action, eq. (22) gets modified so that instead of  $\Phi$ ,  $\Omega = \Phi - 2\epsilon R\sqrt{-g}$  appears. This in turn implies that eq.(23) maintains its form but where  $\sigma$  is replaced by  $\omega = \ln(\frac{\Omega}{\sqrt{-g}}) = \ln(\chi - 2\kappa\epsilon R)$ , where once again,  $\chi = \frac{\Phi}{\sqrt{-g}} = \frac{2U(\phi)}{M+V(\phi)}$ .

Following then the same steps as in the model without the curvature square terms, we can then verify that the connection is the Christoffel symbol of the metric  $\bar{g}_{\mu\nu}$  given by

$$\bar{g}_{\mu\nu} = (\frac{\Omega}{\sqrt{-g}})g_{\mu\nu} = (\chi - 2\kappa\epsilon R)g_{\mu\nu} \quad (40)$$

$\bar{g}_{\mu\nu}$  defines now the "Einstein frame". Equations (39) can now be expressed in the "Einstein form"

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} = \frac{\kappa}{2}T_{\mu\nu}^{eff} \quad (41)$$

where

$$T_{\mu\nu}^{eff} = \frac{\chi}{\chi - 2\kappa\epsilon R}(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\phi_{,\alpha}\phi_{,\beta}\bar{g}^{\alpha\beta}) + \bar{g}_{\mu\nu}V_{eff} \quad (42)$$

where

$$V_{eff} = \frac{\epsilon R^2 + U}{(\chi - 2\kappa\epsilon R)^2} \quad (43)$$

Here it is satisfied that  $\frac{-1}{\kappa}R(\Gamma, g) + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V = M$ , equation that expressed in terms of  $\bar{g}^{\alpha\beta}$  becomes

$$\frac{-1}{\kappa}R(\Gamma, g) + (\chi - 2\kappa\epsilon R)\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V = M. \text{ This allows us to solve for } R \text{ and we get,}$$

$$R = \frac{-\kappa(V + M) + \frac{\kappa}{2}\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\chi}{1 + \kappa^2\epsilon\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi} \quad (44)$$

Notice that if we express  $R$  in terms of  $\phi$ , its derivatives and  $g^{\mu\nu}$ , the result is the same as in the model without the curvature squared term, this is not true anymore once we express  $R$  in terms of  $\phi$ , its derivatives and  $\bar{g}^{\mu\nu}$ .

In any case, once we insert (44) into (43), we see that the effective potential 43 will depend on the derivatives of the scalar field now. It acts as a normal scalar field potential under the conditions of slow rolling or low gradients and in the case the scalar field is near the region  $M + V(\phi) = 0$ .

Notice that since  $\chi = \frac{2U(\phi)}{M+V(\phi)}$ , then if  $M + V(\phi) = 0$ , then, as in the simpler model without the curvature squared terms, we obtain that  $V_{eff} = V'_{eff} = 0$  at that point without fine tuning (here by  $V'_{eff}$  we mean the derivative of  $V_{eff}$  with respect to the scalar field  $\phi$ , as usual).

In the case of the scale invariant case, where  $V$  and  $U$  are given by equation (15), it is interesting to study the shape of  $V_{eff}$  as a function of  $\phi$  in the case of a constant  $\phi$ , in which case  $V_{eff}$  can be regarded as a real scalar field potential. Then from (44) we get  $R = -\kappa(V + M)$ , which inserted in (43) gives,

$$V_{eff} = \frac{(f_1 e^{\alpha\phi} + M)^2}{4(\epsilon\kappa^2(f_1 e^{\alpha\phi} + M)^2 + f_2 e^{2\alpha\phi})} \quad (45)$$

The limiting values of  $V_{eff}$  are:

First, for asymptotically large positive values, ie. as  $\alpha\phi \rightarrow \infty$ , we have  $V_{eff} \rightarrow \frac{f_1^2}{4(\epsilon\kappa^2 f_1^2 + f_2)}$ .

Second, for asymptotically large but negative values of the scalar field, that is as  $\alpha\phi \rightarrow -\infty$ , we have:  $V_{eff} \rightarrow \frac{1}{4\epsilon\kappa^2}$ .

In these two asymptotic regions ( $\alpha\phi \rightarrow \infty$  and  $\alpha\phi \rightarrow -\infty$ ) an examination of the scalar field equation reveals that a constant scalar field configuration is a solution of the equations, as is of course expected from the flatness of the effective potential in these regions.

Notice that in all the above discussion it is fundamental that  $M \neq 0$ . If  $M = 0$  the potential becomes just a flat one,  $V_{eff} = \frac{f_1^2}{4(\epsilon\kappa^2 f_1^2 + f_2)}$  everywhere (not only at high values of  $\alpha\phi$ ). All the non trivial features necessary for a graceful exit, the other flat region associated to the Planck scale and the minimum at zero if  $M < 0$  are all lost. As we discussed in the model without a curvature squared term,  $M \neq 0$  implies that we are considering a situation with S.S.B. of scale invariance.

These kind of models with potentials giving rise to two flat potentials have been applied to produce models for bags and confinement in a very natural way [53]

#### IV. A NOTE ON THE "EINSTEIN" METRIC AS A CANONICAL VARIABLE OF THE THEORY

One could question the use of the Einstein frame metric  $\bar{g}_{\mu\nu}$  in contrast to the original metric  $g_{\mu\nu}$ . In this respect, it is interesting to see the role of both the original metric and that of the Einstein frame metric in a canonical approach to the first order formalism. Here we see that the original metric does not have a canonically conjugated momentum (this turns out to be zero), in contrast, the canonically conjugated momentum to the connection turns out to be a function exclusively of  $\bar{g}_{\mu\nu}$ , this Einstein metric is therefore a genuine dynamical canonical variable, as opposed to the original metric. There is also a lagrangian formulation of the theory which uses  $\bar{g}_{\mu\nu}$ , as we will see in the next section, what we can call the action in the Einstein frame. In this frame we can quantize the theory for example and consider contributions without reference to the original frame, thus possibly considering breaking the TMT structure of the theory through quantum effects, but such breaking will be done "softly" through the introduction of a cosmological term only. Surprisingly, the remaining structure of the theory, reminiscent from the original TMT structure will be enough to control the strength of this additional cosmological term once we demand that the universe originated from a non singular and stable emergent state.

## V. GENERALIZING THE MODEL TO INCLUDE EFFECTS OF ZERO POINT FLUCTUATIONS

The effective energy-momentum tensor can be represented in a form like that of a perfect fluid

$$T_{\mu\nu}^{eff} = (\rho + p)u_\mu u_\nu - p\bar{g}_{\mu\nu}, \quad \text{where} \quad u_\mu = \frac{\phi_{,\mu}}{(2X)^{1/2}} \quad (46)$$

here  $X \equiv \frac{1}{2}\bar{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}$ . This defines a pressure functional and an energy density functional. The system of equations obtained after solving for  $\chi$ , working in the Einstein frame with the metric  $\bar{g}_{\mu\nu}$  can be obtained from a "k-essence" type effective action, as it is standard in treatments of theories with non linear kinetic terms or k-essence models[56]-[59]. The action from which the classical equations follow is,

$$S_{eff} = \int \sqrt{-\bar{g}}d^4x \left[ -\frac{1}{\kappa}\bar{R}(\bar{g}) + p(\phi, R) \right] \quad (47)$$

$$p = \frac{\chi}{\chi - 2\kappa\epsilon R}X - V_{eff} \quad (48)$$

$$V_{eff} = \frac{\epsilon R^2 + U}{(\chi - 2\kappa\epsilon R)^2} \quad (49)$$

where it is understood that,

$$\chi = \frac{2U(\phi)}{M + V(\phi)}. \quad (50)$$

We have two possible formulations concerning  $R$ : Notice first that  $\bar{R}$  and  $R$  are different objects, the  $\bar{R}$  is the Riemannian curvature scalar in the Einstein frame, while  $R$  is a different object. This  $R$  will be treated in two different ways:

1. First order formalism for  $R$ . Here  $R$  is a lagrangian variable, determined as follows,  $R$  that appear in the expression above for  $p$  can be obtained from the variation of the pressure functional action above with respect to  $R$ , this gives exactly the expression for  $R$  that has been solved already in terms of  $X, \phi$ , etc.

2. Second order formalism for  $R$ .  $R$  that appear in the action above is exactly the expression for  $R$  that has been solved already in terms of  $X, \phi$ , etc. The second order formalism can be obtained from the first order formalism by solving algebraically  $R$  from the eq. obtained by variation of  $R$ , and inserting back into the action.

In contrast to the simplified models studied in literature[56]-[59], it is impossible here to represent  $p(\phi, X; M)$  in a factorizable form like  $\tilde{K}(\phi)\tilde{p}(X)$ . The scalar field effective Lagrangian can be taken as a starting point for many considerations.

In particular, the quantization of the model can proceed from (47) and additional terms could be generated by radiative corrections. We will focus only on a possible cosmological term in the Einstein frame added (due to zero point fluctuations) to (47), which leads then to the new action

$$S_{eff,\Lambda} = \int \sqrt{-\bar{g}} d^4x \left[ -\frac{1}{\kappa} \bar{R}(\bar{g}) + p(\phi, R) - \Lambda \right] \quad (51)$$

This addition to the effective action leaves the equations of motion of the scalar field unaffected, but the gravitational equations acquire a cosmological constant. Adding the  $\Lambda$  term can be regarded as a redefinition of  $V_{eff}(\phi, X; M)$

$$V_{eff}(\phi, R) \rightarrow V_{eff}(\phi, R) + \Lambda \quad (52)$$

As we will see the stability of the emerging Universe imposes interesting constraints on  $\Lambda$

After introducing the  $\Lambda$  term, we get from the variation of  $R$  the same value of  $R$ , unaffected by the new  $\Lambda$  term, but as one can easily see then  $R$  does not have the interpretation of a curvature scalar in the original frame since it is unaffected by the new source of energy density (the  $\Lambda$  term), this is why the  $\Lambda$  term theory does not have a formulation in the original frame, but is a perfectly legitimate generalization of the theory, probably obtained by considering zero point fluctuations, notice that quantum theory is possible only in the Einstein frame. Notice that even in the original frame the bar metric (not the original metric) appears automatically in the canonically conjugate momenta to the connection, so we can expect from this that the bar metric and not the original metric be the relevant one for the quantum theory.

## VI. ANALYSIS OF THE EMERGENT UNIVERSE SOLUTIONS

We now want to consider the detailed analysis of The Emerging Universe solutions and in the next section their stability in the TMT scale invariant theory. We start considering the cosmological solutions of the form (in the Einstein frame),

$$ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1-r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right), \phi = \phi(t) \quad (53)$$

in this case, we obtain for the energy density and the pressure, the following expressions. We will consider a scenario where the scalar field  $\phi$  is moving in the extreme right region  $\phi \rightarrow \infty$ , in this case the expressions for the energy density  $\rho$  and pressure  $p$  are given by,

$$\rho = \frac{A}{2}\dot{\phi}^2 + 3B\dot{\phi}^4 + C \quad (54)$$

and

$$p = \frac{A}{2}\dot{\phi}^2 + B\dot{\phi}^4 - C \quad (55)$$

It is interesting to notice that all terms proportional to  $\dot{\phi}^4$  behave like "radiation", since  $p\dot{\phi}^4 = \frac{\rho\dot{\phi}^4}{3}$  is satisfied. here the constants  $A, B$  and  $C$  are given by,

$$A = \frac{f_2}{f_2 + \kappa^2 \epsilon f_1^2}, \quad (56)$$

$$B = \frac{\epsilon \kappa^2}{4(1 + \kappa^2 \epsilon f_1^2 / f_2)} = \frac{\epsilon \kappa^2}{4} A, \quad (57)$$

$$C = \frac{f_1^2}{4 f_2 (1 + \kappa^2 \epsilon f_1^2 / f_2)} = \frac{f_1^2}{4 f_2} A + \Lambda. \quad (58)$$

It will be convenient to "decompose" the constant  $\Lambda$  into two pieces,

$$\Lambda = -\frac{1}{4\kappa^2 \epsilon} + \Delta\lambda \quad (59)$$

since as  $\phi \rightarrow -\infty$ ,  $V_{eff} \rightarrow \Delta\lambda$ . Therefore  $\Delta\lambda$  has the interesting interpretation of the vacuum energy density in the  $\phi \rightarrow -\infty$  vacuum. As we will see, it is remarkable that the stability and existence of non singular emergent universe implies that  $\Delta\lambda > 0$ , and it is bounded from above as well.

The equation that determines such static universe  $a(t) = a_0 = constant$ ,  $\dot{a} = 0$ ,  $\ddot{a} = 0$  gives rise to a restriction for  $\dot{\phi}_0$  that have to satisfy the following equation in order to guarantee that the universe be static, because  $\ddot{a} = 0$  is proportional to  $\rho + 3p$ , we must require that  $\rho + 3p = 0$ , which leads to

$$3B\dot{\phi}_0^4 + A\dot{\phi}_0^2 - C = 0, \quad (60)$$

This equation leads to two roots, the first being

$$\dot{\phi}_1^2 = \frac{\sqrt{A^2 + 12BC} - A}{6B}. \quad (61)$$



The second root is:

$$\dot{\phi}_2^2 = \frac{-\sqrt{A^2 + 12BC} - A}{6B}. \quad (62)$$

It is also interesting to see that if the discriminant is positive, the first solution has automatically positive energy density, if we only consider cases where  $C > 0$ , which is required if we want the emerging solution to be able to turn into an inflationary solution eventually. One can see that the condition  $\rho > 0$  for the first solution reduces to the inequality  $w > (1 - \sqrt{1 - w})/2$ , where  $w = -12BC/A^2 > 0$ , since we must have  $A > 0$ , otherwise we get a negative kinetic term during the inflationary period, and as we will see in the next section, we must have that  $B < 0$  from the stability of the solution, and as long as the discriminant is positive, i.e.  $0 < w < 1$ , it is always true that this inequality is satisfied.

## VII. STABILITY OF THE STATIC SOLUTION

We will now consider the perturbation equations. Considering small deviations of  $\dot{\phi}$  from the static emerging solution value  $\dot{\phi}_0$  and also considering the perturbations of the scale factor  $a$ , we obtain, from Eq. (54)

$$\delta\rho = A\dot{\phi}_0\delta\dot{\phi} + 12B\dot{\phi}_0^3\delta\dot{\phi} \quad (63)$$

at the same time  $\delta\rho$  can be obtained from the perturbation of the Friedmann equation

$$3\left(\frac{1}{a^2} + H^2\right) = \kappa\rho \quad (64)$$

and since we are perturbing a solution which is static, i.e., has  $H = 0$ , we obtain then

$$-\frac{6}{a_0^3}\delta a = \kappa\delta\rho \quad (65)$$

we also have the second order Friedmann equation

$$\frac{1 + \dot{a}^2 + 2a\ddot{a}}{a^2} = -\kappa p \quad (66)$$

For the static emerging solution, we have  $p_0 = -\rho_0/3$ ,  $a = a_0$ , so

$$\frac{2}{a_0^2} = -2\kappa p_0 = \frac{2}{3}\kappa\rho_0 = \Omega_0\kappa\rho_0 \quad (67)$$

where we have chosen to express our result in terms of  $\Omega_0$ , defined by  $p_0 = (\Omega_0 - 1)\rho_0$ , which for the emerging solution has the value  $\Omega_0 = \frac{2}{3}$ . Using this in (65), we obtain

$$\delta\rho = -\frac{3\Omega_0\rho_0}{a_0}\delta a \quad (68)$$

and equating the values of  $\delta\rho$  as given by (63) and (68) we obtain a linear relation between  $\delta\dot{\phi}$  and  $\delta a$ , which is,

$$\delta\dot{\phi} = D_0\delta a \quad (69)$$

where

$$D_0 = -\frac{3\Omega_0\rho_0}{a_0\dot{\phi}_0(A + 12B\dot{\phi}_0^2)} \quad (70)$$

we now consider the perturbation of the eq. (66). In the right hand side of this equation we consider that  $p = (\Omega - 1)\rho$ , with

$$\Omega = 2\left(1 - \frac{U_{eff}}{\rho}\right), \quad (71)$$

where,

$$U_{eff} = C + B\dot{\phi}^4 \quad (72)$$

and therefore, the perturbation of the eq. (66) leads to,

$$-\frac{2\delta a}{a_0^3} + 2\frac{\delta\ddot{a}}{a_0} = -\kappa\delta p = -\kappa\delta((\Omega - 1)\rho) \quad (73)$$

to evaluate this, we use (71), (72) and the expressions that relate the variations in  $a$  and  $\dot{\phi}$  (69). Defining the "small" variable  $\beta$  as

$$a(t) = a_0(1 + \beta) \quad (74)$$

we obtain,

$$2\ddot{\beta}(t) + W_0^2\beta(t) = 0, \quad (75)$$

where,

$$W_0^2 = \Omega_0\rho_0\left[\frac{24B\dot{\phi}_0^2}{A + 12\dot{\phi}_0^2B} - 6\frac{(C + B\dot{\phi}_0^4)}{\rho_0} - 3\kappa\Omega_0 + 2\kappa\right], \quad (76)$$

notice that the sum of the last two terms in the expression for  $W_0^2$ , that is  $-3\kappa\Omega_0 + 2\kappa$  vanish since  $\Omega_0 = \frac{2}{3}$ , for the same reason, we have that  $6\frac{(C+B\dot{\phi}_0^4)}{\rho_0} = 4$ , which brings us to

the simplified expression

$$W_0^2 = \Omega_0 \rho_0 \left[ \frac{24 B \dot{\phi}_0^2}{A + 12 \dot{\phi}_0^2 B} - 4 \right], \quad (77)$$

For the stability of the static solution, we need that  $W_0^2 > 0$ , where  $\dot{\phi}_0^2$  is defined either by E. (61) ( $\dot{\phi}_0^2 = \dot{\phi}_1^2$ ) or by E. (62) ( $\dot{\phi}_0^2 = \dot{\phi}_2^2$ ). If we take E. (62) ( $\dot{\phi}_0^2 = \dot{\phi}_2^2$ ) and use this in the above expression for  $W_0^2$ , we obtain,

$$W_0^2 = \Omega_0 \rho_0 \left[ \frac{4\sqrt{A^2 + 12BC}}{-2\sqrt{A^2 + 12BC} - A} \right], \quad (78)$$

to avoid negative kinetic terms during the slow roll phase that takes place following the emergent phase, we must consider  $A > 0$ , so, we see that the second solution is unstable and will not be considered further.

Now in the case of the first solution, E. (61) ( $\dot{\phi}_0^2 = \dot{\phi}_1^2$ ), then  $W_0^2$  becomes

$$W_0^2 = \Omega_0 \rho_0 \left[ \frac{-4\sqrt{A^2 + 12BC}}{2\sqrt{A^2 + 12BC} - A} \right], \quad (79)$$

so the condition of stability becomes  $2\sqrt{A^2 + 12BC} - A < 0$ , or  $2\sqrt{A^2 + 12BC} < A$ , squaring both sides and since  $A > 0$ , we get  $12BC/A^2 < -3/4$ , which means  $B < 0$ , and therefore  $\epsilon < 0$ , multiplying by  $-1$ , we obtain,  $12(-B)C/A^2 > 3/4$ , replacing the values of  $A, B, C$ , given by (56) we obtain the condition

$$\Delta\lambda > 0, \quad (80)$$

Now there is the condition that the discriminant be positive  $A^2 + 12BC > 0$

$$\Delta\lambda < \frac{1}{12(-\epsilon)\kappa^2} \left[ \frac{f_2}{f_2 + \kappa^2 \epsilon f_1^2} \right], \quad (81)$$

since  $A = \left[ \frac{f_2}{f_2 + \kappa^2 \epsilon f_1^2} \right] > 0$ ,  $B < 0$ , meaning that  $\epsilon < 0$ , we see that we obtain a positive upper bound for the energy density of the vacuum as  $\phi \rightarrow -\infty$ , which must be positive, but not very big.

# VIII. THE VACUUM STRUCTURE OF THE THEORY. EVOLUTION OF THE UNIVERSE, FROM ITS NON SINGULAR ORIGINS TO ITS PRESENT SLOWLY ACCELERATING STATE AT $\phi \rightarrow -\infty$ , CROSSING "BARRIERS".

For the discussion of the vacuum structure of the theory, we start studying  $V_{eff}$  for the case of a constant field  $\phi$ , given by,

$$V_{eff} = \frac{(f_1 e^{\alpha\phi} + M)^2}{4(\epsilon\kappa^2(f_1 e^{\alpha\phi} + M)^2 + f_2 e^{2\alpha\phi})} + \Lambda \quad (82)$$

This is necessary, but not enough, since as we will see, the consideration of constant fields  $\phi$  alone can lead to misleading conclusions, in some cases, the dependence of  $V_{eff}$  on the kinetic term can be crucial to see if and how we can achieve the crossing of an apparent barrier.

For a constant field  $\phi$  the limiting values of  $V_{eff}$  are (now that we added the constant  $\Lambda$ ):

First, for asymptotically large positive values, ie. as  $\alpha\phi \rightarrow \infty$ , we have  $V_{eff} \rightarrow \frac{f_1^2}{4(\epsilon\kappa^2 f_1^2 + f_2)} + \Lambda$ .

Second, for asymptotically large but negative values of the scalar field, that is as  $\alpha\phi \rightarrow -\infty$ , we have:  $V_{eff} \rightarrow \frac{1}{4\epsilon\kappa^2} + \Lambda = \Delta\lambda$ .

In these two asymptotic regions ( $\alpha\phi \rightarrow \infty$  and  $\alpha\phi \rightarrow -\infty$ ) an examination of the scalar field equation reveals that a constant scalar field configuration is a solution of the equations, as is of course expected from the flatness of the effective potential in these regions.

Notice that in all the above discussion it is fundamental that  $M \neq 0$ . If  $M = 0$  the potential becomes just a flat one,  $V_{eff} = \frac{f_1^2}{4(\epsilon\kappa^2 f_1^2 + f_2)} + \Lambda$  everywhere (not only at high values of  $\alpha\phi$ ).

Finally, there is a minimum at  $V_{eff} = \Lambda$  if  $M < 0$ . In summary, and if  $f_2 > 0$ ,  $A > 0$ , we have that there is a hierarchy of vacua ,

$$V_{eff}(\alpha\phi \rightarrow -\infty) = \Delta\lambda < V_{eff}(min, M < 0) = \Lambda < V_{eff}(\alpha\phi \rightarrow \infty) = C \quad (83)$$

where  $C = \frac{f_1^2}{4f_2(1+\kappa^2\epsilon f_1^2/f_2)} + \Lambda = \frac{f_1^2}{4f_2} A + \Lambda$ . notice that we assume above that  $f_1 > 0$  and  $M < 0$ , but  $f_1 < 0$  and  $M > 0$  would be indistinguishable from that situation, that is, the important requirement is  $f_1/M < 0$ . We could have a scenario where we start the non singular emergent universe at  $\phi \rightarrow \infty$  where  $V_{eff}(\alpha\phi \rightarrow \infty) = \frac{f_1^2}{4(\epsilon\kappa^2 f_1^2 + f_2)} + \Lambda$ , which then slow rolls, then inflates [22] and finally gets trapped in the local minimum with energy

density  $V_{eff}(min, M < 0) = \Lambda$ , that was the picture favored in [22], while here we want to argue that the most attractive and relevant description for the final state of our Universe is realized after inflation in the flat region  $\phi \rightarrow -\infty$ , since in this region the vacuum energy density is positive and bounded from above, so its a good candidate for our present state of the Universe. It remains to be seen however whether a smooth transition all the way from  $\phi \rightarrow \infty$  to  $\phi \rightarrow -\infty$  is possible.

In order to discuss the possibility of transition to  $\phi \rightarrow -\infty$ . In our case, since we are interested in a local minimum between  $\phi \rightarrow \infty$  or  $\phi \rightarrow -\infty$ , we can take  $M$  of either sign.

Taking for definitness  $f_1 > 0$ ,  $f_2 > 0$ ,  $A > 0$ ,  $\epsilon < 0$ , we see that there will be a point, defined by  $\epsilon\kappa^2(f_1e^{\alpha\phi} + M)^2 + f_2e^{2\alpha\phi} = 0$  where the effective potential for a constant field  $\phi$ , then  $V_{eff}$  as given by (82), will spike to  $\infty$ , go then down to  $-\infty$  and then asymptotically approach its positive asymptotic value at  $\phi \rightarrow -\infty$ . This has the appearance of a potential barrier. However, this is deceptive, such barrier may exist for constant  $\phi$ , but can be avoided by considering time dependence, say for no space dependence and  $\dot{\phi}^2$  given by

$$\dot{\phi}^2 = -\frac{1}{\epsilon\kappa^2} \quad (84)$$

which has a solution in the real domain for  $\epsilon < 0$ . For this case  $R$  (which is not a Riemannian curvature), as given by (44) diverges. In this case then

$$V_{eff} = \frac{\epsilon R^2 + U}{(\chi - 2\kappa\epsilon R)^2} + \Lambda \rightarrow \frac{1}{4\epsilon\kappa^2} + \Lambda = \Delta\lambda \quad (85)$$

that is, for this value of  $\dot{\phi}^2$ , regardless of the value of the scalar field, the value of  $V_{eff}$  becomes degenerate with its value for constant and arbitrarily negative  $\phi$ , which is our candidate vacuum for the present state of the Universe. Therefore there is no barrier that prevents from us reaching arbitrarily negative  $\phi$  from any point in field space in this model.

## IX. DISCUSSION, THE CREATION OF THE UNIVERSE AS A "THRESHOLD EVENT" FOR ZERO PRESENT VACUUM ENERGY DENSITY

We have considered a non singular origin for the Universe starting from an Einstein static Universe, the so called "emergent universe" scenario, in the framework of a theory which uses two volume elements  $\sqrt{-g}d^4x$  and  $\Phi d^4x$ , where  $\Phi$  is a metric independent density, used

as an additional measure of integration. Also curvature, curvature square terms and for scale invariance a dilaton field  $\phi$  are considered in the action. The first order formalism was applied. The integration of the equations of motion associated with the new measure gives rise to the spontaneous symmetry breaking (S.S.B) of scale invariance (S.I.). After S.S.B. of S.I., using the the Einstein frame metric, it is found that a non trivial potential for the dilaton is generated. One could question the use of the Einstein frame metric  $\bar{g}_{\mu\nu}$  in contrast to the original metric  $g_{\mu\nu}$ . In this respect, it is interesting to see the role of both the original metric and that of the Einstein frame metric in a canonical approach to the first order formalism. Here we see that the original metric does not have a canonically conjugated momentum (this turns out to be zero), in contrast, the canonically conjugated momentum to the connection turns out to be a function exclusively of  $\bar{g}_{\mu\nu}$ , this Einstein metric is therefore a genuine dynamical canonical variable, as opposed to the original metric.

There is also a lagrangian formulation of the theory which uses  $\bar{g}_{\mu\nu}$ , what we can call the action in the Einstein frame. In this frame we can quantize the theory for example and consider contributions without reference to the original frame, thus possibly considering breaking the TMT structure of the theory, but such breaking will be done "softly" through the introduction of a cosmological term only. In previous studies, we have found that the TMT structure of the theory, where neither the lagrangian  $L_1$  that couples to  $\sqrt{-g}$ , or  $L_2$ , that couples to  $\Phi$  depend on the measure fields, is protected by an infinite dimensional symmetry  $\varphi_a \rightarrow \varphi_a + f_a(L_2)$ , where  $f_a(L_2)$  is an arbitrary function of  $L_2$ . The additional cosmological term, introduced here in the Einstein frame, does not have a representation of this form in the original frame, therefore breaking the TMT structure (therefore the infinite dimensional symmetry would be also broken by quantum effects). Surprisingly, the remaining terms of the theory, reminiscent from the original TMT structure will be enough to control the strength of this additional cosmological term once we demand that the universe originated from a non singular and stable emergent state.

In the Einstein frame we argue that the cosmological term parametrizes the zero point fluctuations.

The resulting effective potential for the dilaton contains two flat regions, for  $\phi \rightarrow \infty$  relevant for the non singular origin of the Universe, followed by an inflationary phase and then transition to  $\phi \rightarrow -\infty$ , which in this paper we take as describing our present Universe. An intermediate local minimum is obtained if  $f_1/M < 0$ , the region as  $\phi \rightarrow \infty$  has a higher

energy density than this local minimum and of course of the region  $\phi \rightarrow -\infty$ , if  $A > 0$  and  $f_2 > 0$ .  $A > 0$  is also required for satisfactory slow roll in the inflationary region  $\phi \rightarrow \infty$  (after the emergent phase). The dynamics of the scalar field becomes non linear and these non linearities are instrumental in the stability of some of the emergent universe solutions, which exists for a parameter range of values of the vacuum energy in  $\phi \rightarrow -\infty$ , which must be positive but not very big, avoiding the extreme fine tuning required to keep the vacuum energy density of the present universe small. A sort of solution of the Cosmological Constant Problem, where an a priori arbitrary cosmological term is restricted by the consideration of the nonsingular and stable emergent origin for the universe.

Notice then that the creation of the universe can be considered as a "threshold event" for zero present vacuum energy density, that is a threshold event for  $\Delta\lambda = 0$  and we can learn what we can expect in this case by comparing with well known threshold events. For example in particle physics, the process  $e^+ + e^- \rightarrow \mu^+ + \mu^-$ , has a cross section of the form (ignoring the mass of the electron and considering the center of mass frame,  $E$  being the center of mass energy of each of the colliding  $e^+$  or  $e^-$ ),

$$\sigma_{e^+ + e^- \rightarrow \mu^+ + \mu^-} = \frac{\pi\alpha^2}{6E^2} \left[ 2 + \frac{m_\mu^2}{E^2} \right] \sqrt{\frac{E^2 - m_\mu^2}{E^2}} \quad (86)$$

for  $E > m_\mu$  and exactly zero for  $E < m_\mu$ . We see that exactly at threshold this cross section is zero, but at this exact point it has a cusp, the derivative is infinite and the function jumps as we slightly increase  $E$ . By analogy, assuming that the vacuum energy can be tuned somehow (like the center of mass energy  $E$  of each of the colliding particles in the case of the annihilation process above), we can expect zero probability for exactly zero vacuum energy density  $\Delta\lambda = 0$ , but that soon after we build up any positive  $\Delta\lambda$  we will then be able to create the universe, naturally then, there will be a creation process resulting in a universe with a small but positive  $\Delta\lambda$  which represents the total energy density for the region describing the present universe,  $\phi \rightarrow -\infty$ .

One challenge would be to in fact calculate from this approach the probability of creating the universe with a given vacuum energy density of the vacuum for the region describing the present universe,  $\phi \rightarrow -\infty$ , the same way we calculate the probability of the process  $e^+ + e^- \rightarrow \mu^+ + \mu^-$ . This will give us the probability of a given present vacuum energy density.

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- [1] S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989).
  - [2] Y. Jack Ng, *Int. J. Mod. Phys.* **D1**, 145 (1992).
  - [3] S. Weinberg, astro-ph/0005265.
  - [4] S. Weinberg, *Phys. Rev. Lett.* **59**, 2607 (1987).
  - [5] A.H. Guth, *Phys. Rev.* **D23**, 347 (1981).
  - [6] A.D. Linde, *Phys. Lett.* **108B**, 389 (1982).
  - [7] D. Kazanas, *Astrophys.J.* **241**, 1980 (L59).
  - [8] A. Albrecht and P.J. Steinhardt, *Phys. Rev. Lett.* **48**, 1220 (1982).
  - [9] A.Borde and A.Vilenkin, *Phys. Rev. Lett.* **72**, 3305 (1994).
  - [10] A.Borde and A.Vilenkin, *Phys. Rev.* **D56**, 717 (1997).
  - [11] A.H. Guth, arXiv:astro-ph/0101507.
  - [12] A.Borde, A.H. Guth and A.Vilenkin, *Phys. Rev. Lett.* **90**, 151301 (2003).
  - [13] A.Vilenkin, arXiv:gr-qc/0204061.
  - [14] G.F.R. Ellis and R. Maartens, *Class. Quantum Grav.* **21**, 223 (2004).
  - [15] G.F.R. Ellis, J. Murugan and C.G. Tsagas, *Class. Quantum Grav.* **21**, 233 (2004).
  - [16] D.J. Mulryne, R. Tavakol, J.E. Lidsey and G.F.R. Ellis, *Phys. Rev.* **D71**, 123512 (2005).
  - [17] A. Banerjee, T. Bandyopadhyay and S. Chakraborty, *Grav.Cosmol.* **13**, 290,(2007).
  - [18] S. Mukherjee, B.C.Paul, S.D. Maharaj and A. Beesham, arXiv:qr-qc/0505103.
  - [19] S. Mukherjee, B.C.Paul, N.K. Dadhich, S.D. Maharaj and A. Beesham, *Class. Quantum Grav.* **23**, 6927 (2006).
  - [20] S. del Campo, R. Herrera and P. Labrana, *JCAP* **0907**, 006 (2009).
  - [21] S. del Campo, R. Herrera and P. Labrana, *JCAP* **0711**, 030 (2007).
  - [22] S. del Campo, E.I. Guendelman, R. Herrera and P. Labrana, *JCAP* **1006**, 026 (2010). There are a few typos in this paper that obscure the significance of the results (in particular in what relates the CCP), but the correct corresponding equations appear in this paper, in particular eq. (22) of that paper (the definition of C) has two errors, look at (56) here instead, eq. (40) of that paper contains an error. Some equations where  $\lambda$  (our  $\Lambda$ ) appears should involve a  $\Delta\lambda$  instead (as defined here). Finally, here as opposed to this reference, we take the point of view that the cosmological constant term representing the zero point fluctuations can be

- formulated correctly and unambiguously in the Einstein frame (leading to a consistent effective action) without reference to the original frame, this is because the quantization is performed in the Einstein frame, not in the original frame.
- [23] Basic idea is developed in E.I. Guendelman and A.B. Kaganovich, *Phys. Rev.* **D60**, 065004 (1999).
  - [24] F. Gronwald, U. Muench, A. Macias, F. W. Hehl, *Phys. Rev.* **D58**, 084021 (1998), e-Print: gr-qc/9712063.
  - [25] E.I. Guendelman, A.B. Kaganovich, *Class. Quantum Grav.* **25**, 235015 (2008), e-Print: arXiv:0804.1278 [gr-qc].
  - [26] see also related work by H. Nishino, S. Rajpoot, *Mod. Phys. Lett.* **A21**, 127 (2006), e-Print: hep-th/0404088.
  - [27] D. Comelli, *Int. J. Mod. Phys.* **A23**, 4133 (2008), e-Print: arXiv:0704.1802 [gr-qc] .
  - [28] For a recent review and further references see E.I. Guendelman, A.B. Kaganovich, Plenary talk given at the Workshop on Geometry, Topology, QFT and Cosmology, Paris, France, 28-30 May 2008. e-Print: arXiv:0811.0793 [gr-qc].
  - [29] E.I. Guendelman and A.B. Kaganovich, *Phys. Rev.* **D53**, 7020 (1996).
  - [30] E.I. Guendelman and A.B. Kaganovich, Proceedings of the third Alexander Friedmann International Seminar on Gravitation and Cosmology, ed. by Yu. N. Gnedin, A.A. Grib and V.M. Mostepanenko (Friedmann laboratory Publishing, St. Petersburg, 1995).
  - [31] E.I. Guendelman and A.B. Kaganovich, *Phys. Rev.* **D55**, 5970 (1997).
  - [32] E.I. Guendelman and A.B. Kaganovich, *Mod. Phys. Lett.* **A12**, 2421 (1997).
  - [33] E.I. Guendelman and A.B. Kaganovich, *Phys. Rev.* **D56**, 3548 (1997).
  - [34] E.I. Guendelman and A.B. Kaganovich, *Hadron.Journ.* **21**, 1998 (19).
  - [35] E.I. Guendelman and A.B. Kaganovich, *Mod. Phys. Lett.* **A13**, 1583 (1998).
  - [36] F. Gronwald, U. Muench and F.W. Hehl, *Hadron.Journ.* **21**, 1998 (3).
  - [37] E.I. Guendelman and A.B. Kaganovich, *Phys. Rev.* **D57**, 7200 (1998).
  - [38] E.I. Guendelman and A.B. Kaganovich, "Gravity Cosmology and Particle Field Dynamics without the Cosmological Constant Problem", in the Proceedings of the sixth International Symposium on Particle, Strings and Cosmology, PASCOS-98, World Scientific, Singapore, 1999.
  - [39] E.I. Guendelman and A.B. Kaganovich, "Field Theory Models without the Cosmological Con-

- stant problem”, Plenary talk (given by E.I. Guendelman) on the fourth Alexander Friedmann International Seminar on Gravitation and Cosmology, gr-qc/9809052.
- [40] E.I. Guendelman, *Int. J. Mod. Phys.* **A14**, 3497 (1999).
  - [41] E.I. Guendelman, *Mod. Phys. Lett.* **A14**, 1043 (1999), e-Print: gr-qc/9901017.
  - [42] E.I. Guendelman, *Mod. Phys. Lett.* **A14**, 1397 (1999).
  - [43] E.I. Guendelman, *Class. Quantum Grav.* **17**, 361 (2000).
  - [44] E.I. Guendelman, ”Scale invariance, mass and cosmology” gr-qc/9901067.
  - [45] E.I. Guendelman, ”Scale invariance and the present vacuum energy of the universe”, contribution to the 35-th Rencontres de Moriond, gr-qc/0004011.
  - [46] E.I. Guendelman, ”Measure fields, the cosmological constant and scale invariance” contribution to the 30-th ICHEP conference, hep-th/0008122.
  - [47] E.I. Guendelman, A.B. Kaganovich, *Int. J. Mod. Phys.* **A17**, 417 (2002), e-Print: hep-th/0110040.
  - [48] E.I. Guendelman and A.B. Kaganovich, *Mod. Phys. Lett.* **A17**, 1227 (2002).
  - [49] E.I. Guendelman, A.B. Kaganovich, *Ann. of Phys.* **323**, 866 (2008), e-Print: arXiv:0704.1998 [gr-qc].
  - [50] E.I. Guendelman, A.B. Kaganovich, *Phys. Rev.* **D75**, 083505 (2007), e-Print: gr-qc/0607111.
  - [51] E.I. Guendelman, A.B. Kaganovich, *Int. J. Mod. Phys.* **A21**, 4373 (2006).
  - [52] E.I. Guendelman and O. Katz, *Class. Quantum Grav.* **20**, 1715 (2003), e-Print: gr-qc/0211095.
  - [53] E.I. Guendelman *Int. J. Mod. Phys.* **A25**, 4195 (2010), arXiv:1005.1421 [hep-th].
  - [54] E.I. Guendelman, *Int. J. Mod. Phys.* **A25**, 4081 (2010), e-Print: arXiv:0911.0178 [gr-qc].
  - [55] A. Einstein, ”The Meaning of Relativity”, MJF books, NY (1956), see appendix II.
  - [56] T. Chiba, T. Okabe and M. Yamaguchi, *Phys. Rev.* **D62** 023511 (2000).
  - [57] C. Armendariz-Picon, V. Mukhanov and P.J. Steinhardt, *Phys. Rev. Lett.* **85** 4438, 2000.
  - [58] C. Armendariz-Picon, V. Mukhanov and P.J. Steinhardt, *Phys. Rev.* **D63** 103510 (2001).
  - [59] T. Chiba, *Phys. Rev.* **D66** 063514 (2002).
  - [60] J.D. Barrow and S. Cotsakis, *Phys. Lett.* **214B**, 515 (1988).
  - [61] J.D. Barrow and A.C. Otterwill, *J. Physics* **A16**, 2757 (1983).
  - [62] P.W. Higgs, *Nuovo Cimento* **11** (1959) 816.
  - [63] B. Witt, *Phys. Lett.* **145B**, 176 (1984).
  - [64] If an arbitrary function of the scalar curvature appears see G. Mangano, M. Ferraris and

- M.Francaviglia, *Gen. Rel. Grav.* **19**, 1987 (465) and,
- [65] M.Ferraris, M. Francaviglia and G. Mangano *Class. Quantum Grav.* **5**, L95 (1988).
  - [66] M.B.Mijic, M.M.Morris and W-M Suen, *Phys. Rev.* **D34**, 2934 (1986).
  - [67] The case when there is an additional scalar in the theory has been studied by C.Contreras, R.Herrera and S. del Campo, *Class. Quantum Grav.* **12**, 1937 (1995).
  - [68] The first inflationary model from a model with higher terms in the curvature was proposed in A.A. Starobinsky, *Phys. Lett.* **91B**, 99 (1980).